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ON A CLOSING CRACK†

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A rigorous solution of the mirror-symmetry problem of the closing up of the sides of a crack under the action of a remotely applied compressive load is constructed. The correctness of using the perturbation method to solve a Riemann boundary-value problem when the mapping function differs from a simpler small correction is proved. An asymptotic solution of the problem of a closing crack can be successfully obtained in this way and a second approximation can be constructed for representing the normal stresses on the contour. The use of the perturbation method unexpectedly turns out to be extremely convenient since a sequence of Riemann boundary-value problems have a common matrix coefficient and differ solely in the vectors of the free terms. For this reason, the canonical solution of the homogeneous problem is constructed just once.

1. THE METHOD OF BOUNDARY REPRESENTATIONS

The method of boundary representations of the basic characteristics of the stress–strain state on the contour of a body [1–3], oriented towards simply connected domains which are transformed into canonical rational functions, is convenient for analysing fields of two-dimensional elastic, visco-elastic and viscoplastic media. It consists of the following: any linear combination of basic mechanical quantities on a boundary is a condition of the linear coupling of the boundary values of the analytic functions. For example, the representation

$$\sigma_n = s_1\Omega_1^+ - s_2\Omega_2^+ - s_1\Omega_1^- + s_2\Omega_2^-,\quad 2i\tau_n = s_1\Omega_1^+ + s_2\Omega_2^+ - s_1\Omega_1^- - s_2\Omega_2^-$$

$$2\sigma_\tau = 3s_1\Omega_1^+ + s_2\Omega_2^+ + s_1\Omega_1^- + 3s_2\Omega_2^- \tag{1.1}$$

$$4\mu A\frac{\partial \upsilon_\vartheta}{\partial\theta}\sigma^m|Q|^4 = \kappa A^2\sigma^m\overline{Q}^2\Omega_1^+ + Q^2\Omega_2^+ + A^2\sigma^m\overline{Q}^2\Omega_1^- + \kappa Q^2\Omega_2^-$$

$$\upsilon_\vartheta = \operatorname{Im}[A(u+i\upsilon)],\quad A = e^{-i\vartheta}$$

holds in the case of planar problems in the theory of elasticity, where σ_n, τ_n, σ_τ are the components of the stress tensor, υ_ϑ is the projection of a displacement in a direction which makes an angle ϑ with the y-axis and the quantities s_1, s_2, m are determined by a conformal mapping (ord denotes the order of a polynomial)

$$\omega(\xi) = \frac{Q_0(\xi)}{Q(\xi)},\quad \omega'(\xi) = \frac{s(\xi)}{Q^2(\xi)},\quad s_1 = \frac{1}{s(\sigma)},\quad s_2 = \frac{\sigma^m}{\overline{s(\sigma)}}$$

$$m_0 = \operatorname{ord} Q_0(\xi),\quad m_1 = \operatorname{ord} Q(\xi),\quad m = \max(m_0+m_1-1,\ 2m_1-2) \tag{1.2}$$

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A pair of linear boundary conditions on an arc of the boundary corresponds to the Riemann matrix boundary-value problem

$$\Omega_1^+ = G\Omega_1^- + g, \quad \sigma \in \partial D^+; \quad G = [g_{ij}]_{2\times 2}, \quad g = \{g_1, g_2\} \tag{1.3}$$

The determinant of the matrix-coefficient has a constant modulus, equal to unity as a consequence of the symmetry of the analytic functions

$$\Omega_{1,2}^-(\xi) = \xi^m \overline{\Omega}_{2,1}^+(\xi^{-1}) \tag{1.4}$$

If the boundary of the canonical domain is subdivided into a system of arcs and individual linear boundary conditions are retained on each of the arcs then this, as a whole, constitutes a Riemann boundary-value problem with a discontinuous matrix coefficient with piecewise-rational elements. The distribution laws for mechanical quantities along a boundary of a body can be calculated directly using the formulae of the boundary representation.

It is possible to establish a correspondence between a set of systems of singular integral equations and one and the same Riemann boundary-value problem. The full index of the equations is distinguished from the initial boundary-value problem by the magnitude of the index of the basic problem.

2. FORMULATION OF THE PROBLEM OF A CLOSING CRACK

Let us combine the plan of a physical plane with a crack and the complex plane $z = x + iy$. The contour ∂D of the crack is subdivided into a pair of non-intersecting arcs L_1, L_2 and a point $\gamma \in L_2$ is set in a one-to-one continuous correspondence with any point $t \in L_1$ by means of contact.

The conditions for the contact of the sides are ($U(t)$ is the displacement vector at the point t)

$$\sigma_n(t) + i\tau_n(t) = \sigma_n(\gamma) + i\tau_n(\gamma), \quad U(t) - U(\gamma) = \gamma - t \tag{2.1}$$

Let us consider a class of domains which are mapped onto a circle by the rational function

$$z = \omega(\xi) = A\xi^{-1} + a_1\xi + a_2\xi^2 + \ldots + a_n\xi^n \tag{2.2}$$

By virtue of the requirement that the crack can close up and the constraints on the smallness of the linear and angular displacements, the crevice must have a pair of cuspidal points on the contour and its cross-section must be small compared with its spread.

For one class of such cracks the coefficients can be selected from the conditions that the affices of the points of intersection of the contour with the y-axis have moduli of the order of $\delta << A$, where A is the spread parameter, and the order of the tangency of the sides at the cuspidal points is such as to allow the determination of all the coefficients α_j.

In the simplest case of a bisymmetric crack, the mapping (1.2) has odd real coefficients [4]. In particular, when $n = 3$, the function

$$\omega(\xi) = A_0\left[\left(\frac{1}{2} + \delta\right)\xi^{-1} + \left(\frac{1}{2} - 2\delta\right)\xi + \delta\xi^3\right] \tag{2.3}$$

which maps the contour of a crack with a spread $2A_0$ and a cross-section 4δ onto a circle satisfies all the requirements enumerated above.

In the case of mirror symmetry, problem (2.1) is expressed in the form of a Riemann boundary-value problem

$$\Omega^+ = \frac{1}{\kappa c - \bar{c}} \left\| \begin{matrix} -(c+\bar{c}) & -(\kappa+1)\bar{c}\sigma^{\varepsilon} \\ (\kappa+1)\bar{c}\sigma^{\varepsilon} & \kappa(c+\bar{c}) \end{matrix} \right\| \Omega^- - 2\mu \frac{c+\bar{c}}{\kappa c - \bar{c}} \left\| \begin{matrix} c\sigma^{\alpha-1} \\ -\bar{c}\sigma^{\beta+1} \end{matrix} \right\|, \quad \sigma \in \partial D^+ \tag{2.4}$$

$$c = \sigma\omega'(\sigma), \quad \varepsilon = 2-\alpha+\beta, \quad \alpha = 2, \quad \beta = \max(0, \ n-1)$$

3. THE CONSTRUCTION OF A RIGOROUS SOLUTION

Let us write down the characteristic functions of the matrix coefficient G in Eq. (2.4)

$$h_1 = 1, \quad h_2 = \frac{\kappa\bar{c} - c}{\kappa c - \bar{c}} \tag{3.1}$$

The representation of the matrix G in the canonical Jordan form (R is a polynomial matrix)

$$G = RHR^{-1}, \quad H = \operatorname{diag}\{h_1, \ h_2\}, \quad R(\xi) = \left\| \begin{matrix} -\xi & \xi c(\xi) \\ -\xi^n & \xi^n \bar{c}(\xi^{-1}) \end{matrix} \right\| \tag{3.2}$$

enables us to construct the homogeneous boundary-value problem

$$N^+ = HN^-, \quad N^{\pm}(\xi) = \det R(\xi) R^{-1}(\xi) \Omega^{\pm}(\xi)$$

which is a combination of Riemann problems for each of the piecewise-analytic functions $N^{\pm}_{1,2}(\xi)$ with coefficients 1, h_2 on the periphery ∂D^+. The canonical solution of the first is $x_1^{\pm} = 1$ while, to construct the canonical solution of the second, we decompose the polynomial

$$f(\xi) = \xi^n[\kappa\bar{c}(\xi^{-1}) + c(\xi)] = p^+(\xi)p^-(\xi)p^0(\xi)$$

into factors.

Each of the factors has zeros only in the domains $D^{\pm}$ identified by symbols or on the boundary ∂D^+. The rational function h_2 will then have the representation

$$h_2(\xi) = \xi^r \frac{p^+(\xi)p^-(\xi)}{\bar{p}^-(\xi^{-1})\bar{p}^+(\xi^{-1})} i^{\rho}$$

$$r = \rho + r^+ + r^- = 2n, \quad \rho = \operatorname{ord} p^0(\xi), \quad r^{\pm} = \operatorname{ord} p^{\pm}(\xi)$$

since $f(\xi)$ can only vanish at the stationary points of the mapping which are located on the real axis due to the fact that the coefficients of the mapping are real. Consequently, the representation $p^0(\xi) = (\xi^2 - 1)^k$ holds and, because of this, we obtain

$$p^0(\xi) / \bar{p}^0(\xi^{-1}) \equiv (-1)^k \xi^{2k}, \quad 2k = \rho$$

The index is equal to $2r$ and the canonical solution is given by the formulae [5]

$$x_2^+(\xi) = \xi^r p^-(\xi) / \bar{p}^+(\xi^{-1}), \quad x_2^-(\xi) = i^{\rho}\bar{p}^-(\xi^{-1}) / p^+(\xi) \tag{3.3}$$

The fundamental solution of a homogeneous problem for Eq. (2.4) can be written in the form

$$Z^{\pm}(\xi) = R(\xi) \left\| \begin{matrix} 1 & 0 \\ 0 & x_2^{\pm}(\xi) \end{matrix} \right\| \tag{3.4}$$

(R, $x_2^\pm$ are defined in accordance with (3.2) and (3.3)). We know [6] that, having the fundamental solution, a canonical $X^\pm$ can be constructed for a finite number of steps and that all the operations are replaceable by a single matrix multiplier M which acts on $Z^\pm$ from the right [7]

$$X^\pm = Z^\pm M \tag{3.5}$$

The particular indices of this solution are 0, $2r$. Since the functions $\Omega_{1,2}^\pm$ must be of an order not greater than $n+1$ at infinity, the general solution of problem (2.5) is

$$\Omega^\pm(\xi) = X^\pm(\xi)\left\{\frac{1}{2\pi i}\int_{\partial D^+}\frac{[X^+]^{-1}g}{\sigma-\xi}d\sigma + P(\xi)\right\} \tag{3.6}$$

where a column of the polynomials is characterized by the orders $n+1$, $n+1+2r$.

On account of the fact that the matrix X^+ is rational (since Z^+, M are rational) and the vector g (which is defined in accordance with (2.4)) also has rational components, the integrand can be decomposed into simpler terms and the quadratures carried out in the final form.

A particular solution is picked out when requirements are imposed regarding the symmetry of the functions and the expansion of the Kolosov–Muskhelishvili functions in the neighbourhood of the origin of coordinates in a known form.

The effectiveness of the construction of the solution is therefore predetermined in each actual case. The difficulty is associated with the purely technical question of the decomposition of the polynomials into the simplest factors.

4. ASYMPTOTIC SOLUTION

Even in the relatively simply case of the mapping (2.3), the execution of the operations with respect to the decomposition of the polynomials into their factors and the solution in parametric form (that is, without fixing the numerical values for the parameters A, a_1, a_2, etc.) is problematic.

The use of a small parameter δ enables us to use a perturbation method to solve the Riemann boundary-value problem. Actually, the function (2.3) can be represented in the form

$$\begin{gathered}\omega(\xi) = \omega_0(\xi) + \delta\omega_1(\xi) \\ \omega_0(\xi) = \frac{1}{2}A_0(\xi^{-1}+\xi), \quad \omega_1(\xi) = A_0(\xi^{-1}-2\xi+\xi^3)\end{gathered} \tag{4.1}$$

As can be seen, the function $\omega_0(\xi)$ produces a mapping of the plane into a circle with a cut [−1, 1] along the real axis and the function $\omega_1(\xi)$ introduces a correction to the shape of the domain.

We will show that the use of the perturbation method does not infringe the class of functions which are used to solve the Riemann boundary-value problem.

The asymptotic expansion $U = U_0 + \delta U_1 + \delta^2 U_2 + \dots$ of the solution of a biharmonic equation leads to a series of biharmonic equations $\Delta^2 U_k = 0\,(k = 0, 1, 2, \dots)$.

Each of the functions U_k can be represented in the form of partial complex potentials in accordance with Goursat formula

$$U_k = \operatorname{Re}\{\bar{z}\varphi_k(z) + \chi_k(z)\}$$

and the expansions

$$\varphi=\varphi_0+\delta\varphi_1+\delta^2\varphi_2+\ldots,\quad \chi=\chi_0+\delta\chi_1+\delta^2\chi_2+\ldots$$

are satisfied by virtue of linearity.

Since the basic potentials Φ, Ψ employed to substantiate the boundary representation formulae are expressed in terms of φ, χ in a linear manner, the relations

$$\Phi=\Phi_0+\delta\Phi_1+\delta^2\Phi_2+\ldots,\quad \Psi=\Psi_0+\delta\Psi_1+\delta^2\Psi_2+\ldots$$

also hold for them, where each partial analytic function is expressed in terms of its prototype in a way which is identical to that for the sum as a whole.

Since the parameter δ is real, the form of the functions

$$\overline{\Phi}(\xi^{-1})=\overline{\Phi}_0(\xi^{-1})+\delta\overline{\Phi}_1(\xi^{-1})+\delta^2\overline{\Phi}_2(\xi^{-1})+\ldots$$
$$\overline{\Psi}(\xi^{-1})=\overline{\Psi}_0(\xi^{-1})+\delta\overline{\Psi}_1(\xi^{-1})+\delta^2\overline{\Psi}_2(\xi^{-1})+\ldots$$

which are defined using symmetry, is preserved.

In the case of the meromorphic functions F_1 and F_2 [1–3], in the definition of which there is a transformation function apart from the Kolosov–Muskhelishvili functions (we shall confine ourselves to the case when the function ω consists of two terms, see (4.1)), we have the following asymptotic expressions in which the arguments ξ, ξ^{-1} are omitted for brevity (the argument ξ^{-1} participates in an obligatory order in the functions which are picked by the symbol of complex conjugation)

$$\begin{aligned}
&F_s=F_{s0}+\delta F_{s1}+\delta^2F_{s2}+\ldots\quad (s=1,\ 2)\\
&F_{10}=\omega_0\Phi_0,\quad F_{1k}=\omega_1\Phi_{k-1}+\omega_0\Phi_k\quad (k=1,\ 2,\ \ldots)\\
&F_{20}=-\omega_0'\Phi_0+\xi^2\omega_0\Phi_0'+\xi^2\omega_0'\Psi_0\\
&F_{2k}=-(\omega_1'\Phi_{k-1}+\omega_0'\Phi_k)+\xi^2(\omega_1\Phi_{k-1}'+\omega_0\Phi_k')+\xi^2(\omega_1'\Psi_{k-1}+\omega_0'\Psi_k)\quad (k=1,\ 2,\ \ldots)
\end{aligned}\tag{4.2}$$

Nevertheless, the analytic functions

$$\Omega_s^\pm=\Omega_{s0}^\pm+\delta\Omega_{s1}^\pm+\delta^2\Omega_{s2}^\pm+\ldots\quad (s=1,\ 2)$$

preserve the form in accordance with which they were defined [1–3]

$$\begin{aligned}
&\Omega_{1k}^+=\xi^\alpha F_{1k},\quad \Omega_{2k}^+=\xi^\beta F_{2k},\quad \xi\in D^+\\
&\Omega_{1k}^-=\xi^\alpha \overline{F}_{2k},\quad \Omega_{2k}^-=\xi^\beta \overline{F}_{1k},\quad \xi\in D^-
\end{aligned}\tag{4.3}$$

and each of the functions is of the order of $\alpha+\beta$ at infinity. The general form of the symmetry conditions

$$\Omega_{sk}^-(\xi)=\xi^{\alpha+\beta}\overline{\Omega}_{1-s\ k}^+(\xi^{-1})\quad (s=1,\ 2;\ k=0,\ 1,\ 2,\ \ldots)\tag{4.4}$$

remains true.

We will now consider the conditions on the boundary of a body which are reduced to a Riemann boundary-value problem for two pairs of functions $\Omega^\pm=\{\Omega_1^\pm,\ \Omega_2^\pm\}$

$$\Omega^+=G\Omega^-+g,\quad \sigma\in\partial D^+$$

On expanding the matrix coefficient in an asymptotic series, we can write

$$G=G_0+\delta G_1+\delta^2G_2+\ldots,\quad g=g_0+\delta g_1+\delta^2 g_2+\ldots$$

after which the boundary-value problem is decomposed into a series of problems

$$\Omega_0^+ = G_0\Omega_0^- + g_0, \quad \Omega_1^+ = G_0\Omega_1^- + (g_1 + G_1\Omega_0^-)$$
$$\Omega_2^+ = G_0\Omega_2^- + (g_2 + G_2\Omega_0^- + G_1\Omega_0^-), \ldots \quad (\sigma \in \partial D^+) \tag{4.5}$$

Hence, the use of the perturbation method to solve the linear Riemann boundary-value problem involves the successive construction of the solutions of the partial boundary-value problems (4.5) and the separation of the particular solutions from the general solutions which is subject to the traditional symmetry conditions (4.4) and to the condition that the functions Φ_0, Ψ_0 should be representable in the neighbourhood of the origin of coordinates in the form

$$\Phi_0 = \Gamma - J\xi + \ldots, \quad \Psi_0 = \Gamma' + \kappa\bar{J}\xi + \ldots, \tag{4.6}$$

since the parameter δ does not occur in the coefficients appearing here. We recall that the constants Γ, Γ' specify the stressed state at infinity and the vector J is proportional to the principal vector of the external forces applied to the contour D.

On expanding the coefficients of problem (2.4) in an asymptotic series, in the case of the transformation (2.3) we obtain

$$G_0 = \begin{Vmatrix} 0 & -\bar{\sigma}^2 \\ \sigma^2 & 0 \end{Vmatrix}, \quad g_0 = 0$$
$$G_1 = \frac{c_1 + \bar{c}_1}{\kappa c_0 - \bar{c}_0} \begin{Vmatrix} -1 & -\bar{\sigma}^2 \\ \kappa\sigma^2 & \kappa \end{Vmatrix}, \quad g_1 = 2\mu \frac{c_1 + \bar{c}_1}{\kappa c_0 - \bar{c}_0} \sigma \begin{Vmatrix} -c_0 \\ \sigma^2 \bar{c}_0 \end{Vmatrix} \tag{4.7}$$
$$c = c_0 + \delta c_1, \quad c_0 = \sigma\omega_0', \quad c_1 = \sigma\omega_1'$$

The first of the partial problems (4.5) is homogeneous

$$\Omega_1^+ = -\bar{\sigma}^2\Omega_2^-, \quad \Omega_2^+ = -\sigma^2\Omega_1^-, \quad \sigma \in \partial D^+ \tag{4.8}$$

and, in the class of first-order functions, has the general solution

$$\Omega_{10}^+ = P_0^1 + P_2^1\xi^2, \quad \Omega_{10}^- = -P_0^2\xi^{-2} + P_2^2 + P_4^2\xi^2 + P_6^2\xi^4$$
$$\Omega_{20}^+ = P_0^2 + P_2^2\xi^2 + P_4^2\xi^4 + P_6^2\xi^6, \quad \Omega_{20}^- = -P_0^1\xi^2 - P_2^1\xi^4 \tag{4.9}$$

Here, only the even real coefficients are left (cyclic two-fold symmetry and mirror symmetry; the proof of these cases with respect to Kolosov–Muskhelishvili functions is given in [8]).

The symmetry conditions (4.3) and the representations (4.6), in which $J = 0$ and Γ, Γ' are real on account of the above mentioned symmetry, enable us to conclude that

$$P_2^1 = -P_0^1 = \frac{1}{2}A_0\Gamma, \quad P_0^2 = P_6^2 = 0, \quad P_4^2 = -P_2^2 = -\frac{1}{2}A_0\Gamma'$$

which separates out the particular solution

$$\Omega_{10}^+ = \Gamma f(\xi), \quad \Omega_{20}^+ = \Gamma'\xi^2 f(\xi)$$
$$\Omega_{10}^- = -\Gamma' f(\xi), \quad \Omega_{20}^- = -\Gamma\xi^2 f(\xi), \quad f(\xi) = \frac{1}{2}A_0(\xi^2 - 1) \tag{4.10}$$

The second partial problem (4.5) has the same matrix coefficient as the first. This immediately characterizes it not as a system but as a combination of Riemann problems. After elementary algebra, solely with the participation of the basic approximation which has just been found, its free terms takes the form

$$g_1 + G_1\Omega_0^- = -\frac{3}{\kappa+1}A_0(\sigma^2-1)(\sigma^2-\bar{\sigma}^2)\begin{Vmatrix} 2\mu-(\Gamma'+\Gamma) \\ \sigma^2[2\mu+\kappa(\Gamma'+\Gamma)] \end{Vmatrix} \tag{4.11}$$

The general solution of the problem (here Ω_1 is the vector of the second approximation)

$$\Omega_1^+ = G_1\Omega_1^- + (g_1 + G_1\Omega_0^-), \quad \sigma \in \partial D^+$$

has the form

$$\begin{aligned}
\Omega_{11}^+ &= -b_1(\xi^4-\xi^2-1) + Q_0^1 + Q_2^1\xi^2 \\
\Omega_{21}^- &= -b_1 - Q_0^1\xi^2 - Q_2^1\xi^4, \quad b_1 = 3A_0(2\mu-\Gamma'-\Gamma)/(\kappa+1) \\
\Omega_{21}^+ &= -b_2(\xi^6-\xi^4-\xi^2+1) + Q_0^2 + Q_2^2\xi^2 + Q_4^2\xi^4 + Q_6^2\xi^6 \\
\Omega_{11}^- &= -Q_0^2\xi^{-2} - Q_2^2 - Q_4^2\xi^2 - Q_6^2\xi^4, \quad b_2 = 3A_0(2\mu+\kappa\Gamma'+\kappa\Gamma)/(\kappa+1)
\end{aligned} \tag{4.12}$$

From conditions (4.3) and (4.6), we obtain the coefficients

$$\begin{aligned}
&Q_0^1 = A_0[6\mu-3\Gamma'+(\kappa-2)\Gamma]/(\kappa+1), \quad Q_2^1 = A_0\Gamma \\
&Q_0^2 = 3A_0[2\mu+\kappa\Gamma'+(2\kappa+1)\Gamma]/(\kappa+1), \quad Q_4^2 = A_0(\Gamma'-2\Gamma) \\
&Q_2^2 = -A_0[6\mu-(4\kappa+1)\Gamma'+(\kappa-2)\Gamma]/(\kappa+1), \quad Q_6^2 = -3A_0\Gamma
\end{aligned} \tag{4.13}$$

with which the process of separating out a particular solution in the problem of the second approximation is completed.

The partial boundary-value problem in the next approximation (and so on) has the same matrix coefficient G_0 and, next time also, we do not have a system but a combination of Riemann problems. The process of constructing its solution is simple and only minor difficulties are encountered at the stage of separating out a particular solution and this is solely attributable to the relative complexity of the expressions. We shall confine ourselves to the approximations which have been found and assume that an asymptotic solution is constructed

$$\Omega_s^\pm \approx \Omega_{s0}^\pm + \delta\Omega_{s1}^\pm \tag{4.14}$$

The elastic field can subsequently be constructed using traditional methods. Diagrams of the stresses on the boundary can be constructed starting from the formulae of the boundary representation.

In view of the fact that there are no shear stresses on the boundary, we obtain a simple expression for the normal stress

$$\sigma^2\omega'\sigma_n = \Omega_1^+ - \Omega_1^-, \quad \sigma \in \partial D^+$$

We also expand the normal stress in an asymptotic series

$$\sigma_n = \sigma_{n0} + \delta\sigma_{n1} + \ldots$$

after which, from (4.10), we find that

$$\sigma^2\omega_0'\sigma_{n0} = \Omega_{10}^+ - \Omega_{10}^-, \quad \sigma^2\omega_0'\sigma_{n1} = \sigma^2\omega_1'\sigma_{n0} = \Omega_{11}^+ - \Omega_{11}^-$$

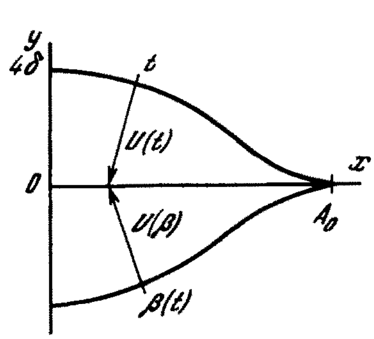

Fig. 1.

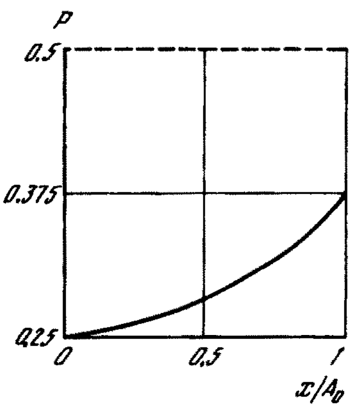

Fig. 2.

On substituting the solution found above, we determine that

$$\sigma_{n0} = \Gamma + \Gamma', \quad \sigma_{n1} = -10\Gamma + 6\frac{2\mu + \kappa\Gamma' + (2\kappa+1)\Gamma}{\kappa+1}(\sigma^2 + \bar{\sigma}^2)$$

and, on transferring to the z-plane, we obtain

$$\sigma_n \approx \Gamma + \Gamma' - \delta\{10\Gamma + 6[2\mu + \kappa\Gamma' + (2\kappa+1)\Gamma]\frac{\kappa^2/A^2 - 2}{\kappa+1}$$

The profile of the crack, which has been substantially expanded along the y-axis for clarity, is shown in Fig. 1. A plot of the normal stresses at the contact is shown in Fig. 2. The first approximation, which corresponds to a homogeneous field at infinity, is shown by the dashed line. It is assumed that a confining compressive stress, P_∞ acts at infinity: $\kappa = 2$, $\mu = 1000 P_\infty$, $A_0 = 1$, $4\mu\delta = P_\infty/8$. The graph of $P = \sigma_n / P_\infty$ has even symmetry.

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